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Short communication

On controllability and system constraints of the linear models of proton exchange membrane and solid oxide fuel cells

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ABSTRACT

In this paper we first show that the linear models of proton exchange membrane (polymer electrolyte membrane, PEM) and solid oxide (SO) fuel cells, commonly used in power and energy literature, are not controllable. The source of uncontrollability is the equation for pressure of the water vapor that is only affected by the fuel cell current, which in fact is a disturbance in this system and cannot be controlled by the given model inputs: inlet molar flow rates of hydrogen and oxygen. Being uncontrollable these models are not good candidates for studying control of dynamic processes in PEM and SO fuel cells. However, due to their simplicity, they can be used in hybrid configurations with other energy producing devices such as photovoltaic (solar) cells, wind turbine, micro gas turbine, battery (ultra capacitor) to demonstrate some other phenomena, but not for control purposes unless the hybrid models formed in such hybrid configurations are controllable. Testing controllability of such hybrid models is mandatory. Secondly, we introduce some algebraic constraints that follow from the model dynamics and the Nernst open-loop fuel cell voltage formula. These constraints must be satisfied in simulation of considered fuel cell modes, for example, via MATLAB/Simulink or any other computer software package.

1. Introduction

The controllability and observability concepts are the system state space concepts. They have been known to control engineers for more than fifty years since the initial work of Kalman [1]. Slowly these concepts are becoming known and used in other engineering and scientific disciplines, especially when the so-called Kalman system canonical decomposition was derived in [2,3]. The Kalman canonical decomposition states that only the system modes that are both controllable and observably appear in the system transfer function and those either uncontrollable or unobservable cancel out from the transfer function (system input/output description). This result has established the fact that the state space system description (via system eigenvalues) is more general than the system description via transfer function (via system poles) since the set of system eigenvalues is broader than the set of system poles (all the poles are the eigenvalues, but not all the eigenvalues are the system poles).

In the first part of the paper we show that the commonly used linear models of PEMFC and SOFC are not controllable. In the second part of the paper we introduce some algebraic constraints on these models that follow from system dynamic equations and from the steady state analysis.

The linear mathematical model, for the PEMFC dynamics of three fundamental fuel cells dynamic variables: pressures of hydrogen, oxygen, and water vapor, was derived in 2004 [4]. The model was obtained by keeping the same state equation and slightly modifying the output equation of the mathematical model derived for the SOFC dynamics in 2000 [5]. These linear mathematical models for PEMFC and SOFC have been used in many papers including some published a year ago, see for example [6–13] for PEMFC related problems and [14–16] for SOFC related problems.

The system state space model given in [4,5] was defined by

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= -\frac{RTK_{H_2}}{V_A}x_1(t) + \frac{RT}{V_A}q_{H_2}^{in}(t) - \frac{2RTK_r}{V_A}I(t) \\
&= -\frac{1}{\tau_{H_2}}x_1(t) + \frac{1}{\tau_{H_2}K_{H_2}}q_{H_2}^{in}(t) - \frac{2K_r}{\tau_{H_2}K_{H_2}}I(t) \\
\frac{dx_2(t)}{dt} &= -\frac{RT}{V_C}K_{O_2}x_2(t) + \frac{RT}{V_C}q_{O_2}^{in}(t) - \frac{RTK_r}{V_C}I(t) \\
&= -\frac{1}{\tau_{O_2}}x_2(t) + \frac{1}{\tau_{O_2}K_{O_2}}q_{O_2}^{in}(t) - \frac{K_r}{\tau_{O_2}K_{O_2}}I(t) \\
\frac{dx_3(t)}{dt} &= -\frac{RT}{V_C}K_{H_2O}x_3(t) + \frac{2RTK_r}{V_C}I(t) \\
&= -\frac{1}{\tau_{H_2O}}x_3(t) + \frac{2K_r}{\tau_{H_2O}K_{H_2O}}I(t)
\end{aligned} \tag{1}$$

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with the state space variables representing

$$x(t) = [\, x_1(t) \quad x_2(t) \quad x_3(t) \,]^T = [\, p_{H_2}(t) \quad p_{O_2}(t) \quad p_{H_2O}(t) \,]^T \tag{2}$$

The output equation represents the measured fuel cell voltage and it is obtained using the Nernst formula for the open-loop cell voltage, $V_0(t)$, and subtracting losses due to the cell activation, $V_{act}(t)$, and due to the stack (fuel cell) resistance, $V_{ohm}(t)$

$$\begin{aligned} y_{PEM}(t) &= V_{PEM}(t) = V_0(t) - V_{act}(t) - V_{ohm}(t) \\ &= N\left(E_0 + \frac{RT}{2F}\ln\left\{\frac{x_1(t)(x_2(t))^{0.5}}{x_3(t)}\right\}\right) \\ &\quad -B\ln(CI(t)) - R^{\text{int}}I(t) \end{aligned} \tag{3}$$

The system inputs are molar flow rates of hydrogen and oxygen, that is, $q_{H_2}(t)$ and $q_{O_2}(t)$ that can be regulated (controlled). The stack current $I(t)$ plays a role of a disturbance. Note that $CI(t)$ must be greater than 1, otherwise the activation voltage will be negative, and hence it will increase the open-loop voltage (instead of reducing it). All other coefficients are assumed to be constant. The values of the constant coefficients defined in the model equations can be found in [4].

The SOFC fuel cell model of [5] has exactly the same state Eqs. (1) and (2), but different output equation in which the activation voltage is not present in the expression for the cell output voltage, that is

$$y_{SO}(t) = V_{SO}(t) = N\left(E_0 + \frac{RT}{2F}\ln\left\{\frac{x_1(t)(x_2(t))^{0.5}}{x_3(t)}\right\}\right) - R^{\text{int}}I(t) \tag{4}$$

Of course, in the SOFC mathematical model (1), (2) and (4) the parameters take different values (except for the universal gas constant R and the Faraday constant F). The values of the constant parameters for the SOFC model defined by (1) and (4) can be found in [5].

2. Controllability of linear PEMFC and SOFC models

The importance of controllability in the design of linear controllers for PEM fuel cells was nicely demonstrated in [17], where even for originally controllable operating points of a linearized system some design techniques provide high controllability measures (requiring less control efforts and more efficient control) than the other also controllable operation points. Controllability analysis of liquid water in a fuel cell has been considered in a very recent paper [18], where it has been concluded that liquid water controllability is needed to prevent the fuel cell flooding. In this brief note we will show that the models of [4,5] are uncontrollable (zero controllability measure), meaning that no control efforts exist to satisfy general goals of transferring state variable from a given initial state to a desired final state in a finite time interval [3].

The state space model (1) can be represented in the state space form as

$$\begin{aligned} \frac{dx(t)}{dt} &= \begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \\ \frac{dx_3(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{1}{\tau_{H_2}} & 0 & 0 \\ 0 & -\frac{1}{\tau_{O_2}} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} \frac{1}{\tau_{H_2}K_{H_2}} & 0 \\ 0 & \frac{1}{\tau_{O_2}K_{O_2}} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} q_{H_2}^{in}(t) \\ q_{O_2}^{in}(t) \end{bmatrix} + \begin{bmatrix} -\frac{2K_r}{\tau_{H_2}K_{H_2}} \\ -\frac{K_r}{\tau_{O_2}K_{O_2}} \\ \frac{2K_r}{\tau_{H_2O}K_{H_2O}} \end{bmatrix} I(t) \\ &= Ax(t) + Bu(t) + Gd(t) \end{aligned} \tag{5}$$

where $u(t) = [\, q_{H_2}^{in}(t) \quad q_{O_2}^{in}(t) \,]^T$ are the control inputs and $d(t) = I(t)$ denotes the system disturbance. Using the standard controllability test [3], we can form the controllability matrix for the state space system defined in (5) given by

$$\begin{aligned} C(A,B) &= \begin{bmatrix} B & AB & A^2B \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{\tau_{H_2}K_{H_2}} & 0 & -\frac{1}{\tau_{H_2}^2K_{H_2}} & 0 & \frac{1}{\tau_{H_2}^3K_{H_2}} & 0 \\ 0 & \frac{1}{\tau_{O_2}K_{O_2}} & 0 & -\frac{1}{\tau_{O_2}^2K_{O_2}} & 0 & \frac{1}{\tau_{O_2}^3K_{O_2}} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{6}$$

It is obvious that the rank of the controllability matrix $C(A, B)$ is equal to 2, that is

$$\text{rank}\{C(A,B)\} = 2 < 3 = n \tag{7}$$

This indicates that in the third-order dimensional linear system considered only two state variables are controllable, and the third one is uncontrollable. Examining the state space equations, it can be observed that the equation for the water vapor is not affected by the control input signal and hence, the water vapor pressure is the uncontrollable variable in this system.

The system controllability in this particular model is needed for several reasons. First of all, it is well known that being uncontrollable, the state variable $x_3(t)$ will not appear in the system transfer function [3], which in this case means that the system transfer function is of order two, corresponding to the controllable state variables $x_1(t)$ and $x_2(t)$. Hence, every frequency domain analysis that involves model (1) will be superficial since it will not involve the state variable $x_3(t)$. Secondly, it is known from [3] that state feedback can be used to stabilize unstable systems, but it can not make uncontrollable systems controllable so that the variable $x_3(t)$ by no means can be affected by control input signals, and it will remain affected only by the disturbance signal $I(t)$ that changes randomly as $I(t) = V_{fc}(t)/R_L$, where the load R_L changes randomly in time as a piecewise constant. Hence, changes in the dynamics of the state variable $x_3(t)$ will be fully determined only by its time constant and the fuel cell disturbance (current). It should be emphasized that according to the numerical data from [4] and [5] the time constant for $x_3(t)$ is much larger than for the remaining two state variables ($\tau_{H_2O} = 18.418\,\text{s}$, $\tau_{O_2} = 6.64\,\text{s}$, $\tau_{H_2} = 3.37\,\text{s}$ for PEMFC [4], and $\tau_{H_2O} = 78.3\,\text{s}$, $\tau_{O_2} = 2.91\,\text{s}$, $\tau_{H_2} = 26.1\,\text{s}$ for SOFC [5]) which means that $x_3(t)$ takes much longer time to reach its steady state value (when it will be dictated only by the steady state value of the current $x_3^{ss} = 2K_rI^{ss}/K_{H_2O}$) than the remaining two state variables. Moreover, the magnitude of the state variable $x_3(t)$ is much smaller than the magnitude of the state variables $x_1(t)$ and $x_2(t)$, and since it appears in the dominator of the cell output voltage formula (4), it will have a more dominant, more lasting, and more unpredictable impact on the cell output voltage. Thirdly, controlling water in a fuel cell is fundamentally important [18], since it can cause cell flooding, degrade the cell polarization curve, and eventually damage the cell membrane [19,20] (note that the water vapor mass m_{H_2O} is proportional to the water vapor pressure p_{H_2O} [19]).

3. System analysis constraints of PEMFC and SOFC models

In this section, we derive some algebraic constraints that follow from the model differential equations. These constraints were not imposed in the papers [4,5] that derived the considered models nor in any other follow-up paper that have used these models alone or in hybrid configurations with other electric energy generating devices. The constraints are imposed at steady state, for the initial conditions, and for all time instants.

3.1. Steady state constraints

The steady state value of the water vapor (obtained by setting the derivative to zero and solving the corresponding algebraic equation) is given by

$$x_3^{ss} = \frac{2K_r}{K_{H_2O}} I^{ss} \tag{8}$$

The steady state values of the hydrogen and oxygen pressures are functions of the steady state values of the molar inlet rates of hydrogen and oxygen and the steady state fuel cell current. They are respectively given by

$$x_1^{ss} = \frac{1}{K_{H_2}} q_{H_2 ss}^{in} - \frac{2K_r}{K_{H_2}} I^{ss}, \quad x_2^{ss} = \frac{1}{K_{O_2}} q_{O_2 ss}^{in} - \frac{K_r}{K_{O_2}} I^{ss} \tag{9}$$

The corresponding steady state output voltage value (for SOFC) from (4) is given by

$$\begin{aligned} y_{SO}^{ss} &= V_{SO}^{ss} = N\left(E_0 + \frac{RT}{2F}\ln\left\{\frac{x_1^{ss}(x_2^{ss})^{0.5}}{x_3^{ss}}\right\}\right) - R^{\text{int}} I^{ss} \\ &= N\left(E_0 + \frac{RT}{2F}\ln\left\{\frac{K_{H_2O}}{2K_r K_{H_2}\sqrt{K_{O_2}}} \right.\right. \\ &\quad \left.\left. \times \frac{(q_{H_2 ss}^{in} - 2K_r I^{ss})(q_{O_2 ss}^{in} - K_r I^{ss})^{0.5}}{I^{ss}}\right\}\right) - R^{\text{int}} I^{ss} \end{aligned} \tag{10}$$

Since the hydrogen and oxygen pressures are positive quantities, the following conditions on the inlet hydrogen and oxygen flow rates must be satisfied at the steady state

$$\begin{aligned} x_1^{ss} &= \frac{1}{K_{H_2}} q_{H_2 ss}^{in} - \frac{2K_r}{K_{H_2}} I^{ss} > 0 \Rightarrow q_{H_2 ss}^{in} > 2K_r I^{ss} \\ x_2^{ss} &= \frac{1}{K_{O_2}} q_{O_2 ss}^{in} - \frac{K_r}{K_{O_2}} I^{ss} > 0 \Rightarrow q_{O_2 ss}^{in} > K_r I^{ss} \end{aligned} \tag{11}$$

Another set of simulation constraints comes from the Nernst open-loop voltage that is a positive quantity. Since the "ln" operation is present in that formula we must have

$$\ln\left\{\frac{x_1^{ss}(x_2^{ss})^{0.5}}{x_3^{ss}}\right\} > 0 \Rightarrow \frac{x_1^{ss}(x_2^{ss})^{0.5}}{x_3^{ss}} > 1 \Rightarrow x_1^{ss}(x_2^{ss})^{0.5} > x_3^{ss} \tag{12}$$

which implies

$$(q_{H_2 ss}^{in} - 2K_r I^{ss})(q_{O_2 ss}^{in} - K_r I^{ss})^{0.5} > \frac{2K_r K_{H_2}\sqrt{K_{O_2}}}{K_{H_2O}} I^{ss} \tag{13}$$

Hence, the steady state hydrogen and oxygen pressures must be such to overcome not just the constraint given in (11), but also the stronger constraint given in (13), which is dictated by the cell (stack) steady state current.

3.2. Initial condition and time constraints

The constraints analogous to the steady state constraints imposed in (12) and (13) must be extended for all time instances. This due to the facts that the pressures are positive quantities at all times, that is, $x_1(t) > 0$, $x_2(t) > 0$, $x_3(t) > 0$, and that from the Nernst formula we must have

$$\frac{x_1(t)(x_2(t))^{0.5}}{x_3(t)} > 1, \forall t \Rightarrow x_1(t)(x_2(t))^{0.5} > x_3(t), \forall t \tag{14}$$

The pressure positivity requirements will also require imposing constraints on the hydrogen and oxygen pressure initial conditions.

The analytical expression for the hydrogen and oxygen pressures and corresponding initial condition constraints are given by

$$\begin{aligned} x_1(t) &= e^{-\frac{1}{\tau_{H_2}}t} x_1(0) + \int_0^t e^{-\frac{1}{\tau_{H_2}}(t-\tau)}\left(q_{H_2}^{in}(\tau) - \frac{2K_r}{\tau_{H_2}K_{H_2}} I(\tau)\right)d\tau > 0, \forall t \\ &\Rightarrow x_1(0) > -\int_0^t e\frac{1}{\tau_{H_2}}\tau\left(q_{H_2}^{in}(\tau) - \frac{2K_r}{\tau_{H_2}K_{H_2}} I(\tau)\right)d\tau \end{aligned} \tag{15}$$

and

$$\begin{aligned} x_2(t) &= e^{-\frac{1}{\tau_{O_2}}t} x_2(0) + \int_0^t e^{-\frac{1}{\tau_{O_2}}(t-\tau)}\left(q_{O_2}^{in}(\tau) - \frac{K_r}{\tau_{O_2}K_{O_2}} I(\tau)\right)d\tau > 0, \forall t \\ &\Rightarrow x_2(0) > -\int_0^t e\frac{1}{\tau_{O_2}}\tau\left(q_{O_2}^{in}(\tau) - \frac{K_r}{\tau_{O_2}K_{O_2}} I(\tau)\right)d\tau \end{aligned} \tag{16}$$

It is interesting to observe the initial condition constraints on $x_1(0)$ and $x_2(0)$ imposed in (15) and (16). Even though, the coefficients that multiply the stack current in (15) and (16) are small (in [4] they are 0.02 and 0.01, respectively for hydrogen and oxygen) these initial condition constraints must be satisfied in simulation studies whenever these models are used, otherwise in the initial time interval the negative values could be obtained for the hydrogen and/or oxygen pressures (using MATLAB/Simulink or any other simulation software package). There is no similar initial condition constraint on the water vapor, except for the obvious one that comes from the Nernst formula $x_3(0) \neq 0$ (also, the same formula require in general $x_3(t) \neq 0, \forall t$). The water vapor values in time as a function of the fuel cell current are obtained by solving the last equation in (1), leading to

$$x_3(t) = e^{-\frac{1}{\tau_{H_2O}}t} x_3(0) + \frac{2K_r}{\tau_{H_2O}K_{H_2O}} \int_0^t e^{-\frac{1}{\tau_{H_2O}}(t-\tau)} I(\tau)d\tau \tag{17}$$

clearly indicating that the expression for the water pressure is not a function of the inlet molar flow rates of the hydrogen and oxygen.

Since the cell current depends on the load R_L that changes in time independently outside of the fuel cell (as a piecewise constant function), with $V_{fc}(t) = R_L I(t)$, the cell current is in general variable and the above stated constraints, (11) and (13)–(16) must hold for the worse case scenario (when $I(t)$ takes its maximal values, that is, for $I_{max} = \max\{I(t)\}, \forall t$).

4. Conclusions

The lack of controllability of considered linear models of PEM and SO fuel cells might mean that the results presented in journal and conference papers using these uncontrollable models might be valid only for the chosen set of data, and that the conclusions drawn in those papers are not general (valid for all possible inputs and all possible values of the state space variables). We have also introduced some algebraic constraints that must be satisfied at steady state, initial time, and at all times to be able to run simulations (in MATLAB/Simulink or any other computer software package) of the considered linear PEM and SO fuel cell models.

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